J. Fluid Mech. (1973), *vol.* 57, *part* 4, *pp*, 625–649
Printed in Great Britaib

Report on the NATO Advanced Study Institute on magnetohydrodynamic phenomena in rotating fluids

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(Received 27 October 1972)

An Advanced Study Institute sponsored by NATO on "Magnetohydrodynamic Phenomena in Rotating Fluids" was held in Cambridge, England, from 26 June to 4 July 1972. The aim of the meeting was to provide, through invited lectures, a systematic account of those aspects of the geophysics of the earth's core and of astrophysics in which both rotation effects and magnetic effects play an important part. In addition to the invited lectures, shorter contributions and discussions were strongly encouraged. The main Sessions and the Session Chairmen were as follows:

A	The earth's magnetism and planetary magnetism	Sir Edward Bullard
B	Convection, differential rotation, and magnetohydrodynamics in the sun	Dr N. O. Weiss
C	Stellar and interstellar magnetism; pulsars	Prof. E. Spiegel
D	Boundary layers and detached shear layers and spin-up problems	Prof. E. R. Benton
E	Waves and instabilities influenced by Lorentz and/or Coriolis forces	Dr R. Hide
F	Dynamo theory	Prof. P. H. Roberts

These Chairmen, together with Professor P. A. Gilman and Dr H. K. Moffatt, who acted as Director of the Institute, constituted the Organising Committee. The summary of the meeting which follows has been substantially written by Dr Weiss (sessions B and C), Dr Hide (A and E), Professor Benton (D) and Professor Roberts (F), whose willing collaboration is gratefully acknowledged.

There were just over a hundred participants, the great majority of whom were accommodated at Trinity College, Cambridge, for the duration of the meeting.

1. The earth's magnetism and planetary magnetism

The earth and Jupiter possess fairly strong global-scale magnetic fields, the evidence in the case of Jupiter being non-thermal radio-emission at decimetre and dekametre wavelengths. A weak and patchy magnetic field has recently been found at the surface of the Moon. Space-probe magnetometer measurements have been made in the vicinity of Venus and Mars, with essentially negative results. There is no evidence, radio-astronomical or otherwise, that Saturn, Uranus and Neptune are magnetic, but as these are large, rapidly rotating, and partially or wholly fluid planets it is not unlikely that fields will be detected when space-probe measurements are eventually made.

Sir Edward Bullard† opened Session A with a survey of observations of the earth's magnetic field, which is subject to irregular temporal variations on time scales ranging from fractions of a second up to tens or even hundreds of millions of years. The main geomagnetic field, which has its origin in the liquid core of the earth, can at the present time be roughly represented by the field of a hypothetical centred and nearly axial dipole with a moment 8×10^{25} e.m.u., giving a surface field of about 0·5 gauss. The non-dipole part of the main field is a good deal weaker at the earth's surface than the dipole field ($\sim$ 20 per cent) but it fluctuates more rapidly and the r.m.s. values of the respective contributions of these two parts of the field to the geomagnetic secular variation are roughly comparable. A striking feature of the geomagnetic secular variation is the tendency for the non-dipole field to drift westward at about 0·2° of longitude per year, which is evidently much faster than any westward drift of the dipole field. At the core–mantle interface the spherical harmonic coefficients of the field fall off rather more rapidly than white noise.

Studies of the magnetization of rocks have extended our knowledge of the earth's magnetic field back over geological time. The most remarkable result of this work is the discovery that the polarity of the dipole field has changed in a complicated and erratic way. Thus while two dozen polarity reversals occurred during the past $4{\cdot}5 \times 10^6$ years, the polarity remained unchanged for as long an interval as 30×10^6 years during the Permian (Bullard 1968). With only a few thousand years as a typical interval of time taken for the dipole moment to change sign by first decreasing in magnitude while remaining roughly axial and then increasing as the new polarity is established, individual reversals are comparatively sudden events. Two particularly puzzling observational results are (*a*) an apparent correlation in certain rocks between magnetic polarity and state of oxidation, and (*b*) the extinction of certain species at the last reversal.

Another property of reversals was dealt with by *M. Kono* later in the session when he described some stochastic models of the earth's magnetic field during the past 10^7 years. Palaeomagnetic researches have established that during this period the geomagnetic field had the following properties: (*a*) it was essentially a dipolar field with its axis not much removed from the present rotation axis; (*b*) that the observed frequency of the geomagnetic dipole intensities show a Gaussian distribution; and (*c*) that the lengths of the polarity intervals show a Poisson distribution. These properties can evidently be described in terms of a model involving twenty or so hypothetical dipoles scattered throughout the liquid core of the earth.

Representing the earth's magnetism in terms of hypothetical dipoles within the core was first attempted more than twenty years ago, as a proposed improvement on the traditional representation in terms of spherical harmonics. Though useful for certain purposes, the method can be physically misleading and when combined with the skin depth arguments appropriate to a solid conductor it led to the erroneous idea that the geomagnetic secular variation must have its origin

† Those who contributed lectures at the meeting are distinguished by italic type.

within the top few tens of kilometres of the core. Such arguments do not, of course, apply to a conducting *fluid*, through which magnetic energy can be transmitted much more efficiently by fluid motions.

Thus, early work on hydromagnetic waves in the earth was concerned with the upward transmission of magnetic energy in the core and across the core–mantle interface. Several papers on the reflexion and refraction of plane, small amplitude hydromagnetic waves at a plane rigid surface have now been written with the problem of the geomagnetic secular variation in mind. *D. D. Skiles* reviewed these papers, pointing out that incorrect boundary conditions had been used in early work, and presented the results of his own recent analysis (Skiles 1972). Work is now in progress on geophysically important effects due to rotation, non-uniformity of the basic fields, and the shape of the core–mantle interface.

It is now generally accepted that (*a*) the main geomagnetic field is due to ordinary electric currents within the earth and (*b*) these currents flow mainly in the liquid core, where they are generated by a hydromagnetic dynamo process involving fluid motions of a fraction of a millimetre per second. Thus, the problem of the earth's magnetism is inextricably linked with that of providing a theory of core motions, and this in turn will be inseparable from developments in the hydromagnetics of rotating fluids. *R. Hide* reviewed this part of the subject, pointing out that high on the list of geophysical phenomena to be accounted for by an acceptable theory of core motions are (i) the highly variable frequency of reversals in sign of the geomagnetic dipole and (ii) the comparatively short time scale of irregular variations in the length of the day.

The magnetic flux linkage of a perfect conductor cannot change, so that effects due to ohmic dissipation of energy are central to dynamo theories, the subject of Session F of this conference (see § 6). However, on the short time scale of the geomagnetic secular variation, the main body of the core should behave in many respects like a perfect conductor and support free hydromagnetic oscillations of various kinds if they can be excited. If the strength of the toroidal magnetic field in the core is $\sim$ 100 gauss, then those planetary scale oscillations which are characterized by 'magnetostrophic' balance between Coriolis and Lorentz torques on individual fluid elements would have periods comparable with the time scale of the geomagnetic secular variations and they might propagate in a manner reminiscent of the westward drift. These slow oscillations might also play a key role in the production of the geomagnetic field by the dynamo process. (For reviews and extensive lists of references see Acheson & Hide 1972; Hide & Stewartson 1972; Roberts & Soward 1972.)

There is another general class of planetary-scale hydromagnetic oscillations of the core but these have periods that are typically much less than the electromagnetic cut-off period of a few years associated with the weak conductivity of the earth's mantle and they are consequently incapable of producing magnetic effects at the earth's surface. However, the eddy currents induced in the lower mantle by these fast oscillations might suffice to account for the horizontal stresses at the core–mantle interface that produce irregular changes in the earth's rotation. An alternative suggestion as to the nature of these horizontal stresses is that they are largely due to hydromagnetic interactions between core motions and bumps on the core–mantle boundary with typical horizontal dimensions up to a few thousand kilometres and vertical dimensions of only a few kilometres. Such bumps cannot yet be resolved by seismological methods but evidence of their existence is provided by the recently discovered correlation between planetary scale features of the earth's gravitational and magnetic fields (Hide & Malin 1970).

The complexity of the horizontal structure (i.e. temperature, height, etc.) of the core–mantle interface could determine the frequency of reversals in sign of the geomagnetic dipole and other properties of the field, such as the amplitudes of the non-dipole field in secular variation (for review see Cox & Cain 1972). Such effects will have to be taken into account when formulating realistic models of core motions, a procedure which will also require further basic studies of the hydromagnetics of rotating fluids in containers of various shapes. One flow property to which particular attention will be paid in these studies is the helicity (the scalar product of the velocity and vorticity field), the importance of which emerges from the work of Parker and others on kinematic dynamos. A surprisingly simple relationship between helicity and local heat transfer can be found by combining the equations of motion of a rapidly rotating fluid with the equation of thermodynamics (Hide, unpublished).

The agencies that stir the core have not yet been identified with certainty, owing largely to ignorance of the detailed chemical composition and other properties of the earth's deep interior, especially the radioactivity of the core. In reviewing in some detail the suggestions that mechanical stirring associated with the precession of the earth's rotation axis and thermal stirring due to radioactive heating or to the release of heat of crystallization might suffice, *W. V. R. Malkus* outlined some of his own important fluid-dynamical experiments on precession and recent studies of convection at the melting point (Malkus 1971).

The magnetism of the earth took pride of place in Session A, but included in Hide's lecture was a brief discussion of the magnetic fields of Jupiter and the moon. Variability in the rotation of the Jovian radiosources and of the Great Red Spot has been interpreted as evidence of a gross torsional oscillation of Jupiter's internal layers involving a toroidal magnetic field of over 10^3 gauss within the planet. The electric currents responsible for this field and for the comparatively weak poloidal field of 10 gauss at the visible surface are probably produced by hydromagnetic dynamo action driven by thermal convection in Jupiter's metallic core, if it is liquid, or even in Jupiter's lower atmosphere, if it is deep enough, the convection being maintained by gravitational energy release associated with slow contraction in radius of the planet at about 0·1 cm per year (for review see Hide 1971*a*).

Permanent magnetization of lunar rocks is evidently responsible for the weak ($\sim 10^{-3}$ gauss) and patchy magnetic field found at the moon's surface. Some authors account for this permanent magnetization by invoking a hydromagnetic dynamo in a hypothetic lunar core in order to account for an inducing field of not less than 10^{-2} gauss, while others feel that alternative sources of the inducing field, such as the electric current generated by induction during meteorite impact, have not yet been ruled out and might prove to be important. (For pertinent references see various articles in *The Moon*, **4**, 1–268, 1972.)

2. Convection, differential rotation, and magnetohydrodynamics in the sun

The sun's magnetic field has been described in two recent reviews (Parker 1970*a*; Weiss 1971). The main observational features were summarized by *N. O. Weiss*. The solar cycle is shown most clearly by the behaviour of toroidal fields, emerging in bipolar magnetic regions or sunspots and reversing after 11 years. The distribution of these fields provides the characteristic butterfly diagram. The poloidal field can be observed at high latitudes, and reverses, rather erratically, around the time of sunspot maximum. Near the equator, the general field shows a sector structure: activity occurs in preferred latitudes which rotate with a fixed synodic period of 27 days, which can be detected in the interplanetary magnetic field and in recurrent magnetic storms (Wilcox 1971). Detailed measurements show a much more complicated structure, with fields varying on scales down to 1500 km, while the average field over the whole sun fluctuates from day to day in much the same way as the fields measured in stars with convective envelopes like Sirius and Vega (Severny 1971).

A number of dynamo models have been proposed to explain the solar cycle. All depend on the mechanism first put forward by Parker (1955), whereby the poloidal field is drawn out by differential rotation to give a toroidal field from which a poloidal field with the opposite sense is produced by cyclonic motions. Babcock (1961) suggested a phenomenological model which was elaborated by Leighton (1969), who integrated the dynamo equations and obtained a butterfly diagram.

F. Krause and *K. H. Rädler* described a more rigorous treatment (Steenbeck & Krause 1969 – see Roberts & Stix 1971; Krause & Rädler 1971) based on mean field electrodynamics. The averaged vector product of the fluctuations in the velocity and the magnetic field provides an electromotive force parallel to the mean magnetic field (the α-effect), arising from inhomogeneous turbulence influenced by Coriolis forces. By using Bochner's theorem it can also be shown that there is generally an enhanced turbulent resistivity (Krause & Roberts 1973). In the sun, the radial density gradient forces rising gas (which dominates the motion) to expand and so to rotate, clockwise in the northern hemisphere and anti-clockwise in the south. Thus there is a net helicity, or α-effect. Solar dynamos based on this α-effect plus a radial shear in the angular velocity produce butterfly diagrams in excellent agreement with the observations. These calculations have been elaborated by others (Deinzer & Stix 1971; Roberts & Stix 1972; Stix 1972) who show that migrations of sunspot zones towards the equator requires an angular velocity that decreases with radius. *H. Köhler* described a solution of the dynamo equations in the solar convective zone, using finite differences to obtain a solution matched to a potential field outside. Both α and the turbulent diffusivity are calculated from a model of the convective zone. Köhler pointed out that an oscillatory dynamo could only be obtained if the value of α was several orders of magnitude less than that predicted by Steenbeck & Krause (1969).

The α-effect relies on an idealized model of turbulence and it is desirable to relate dynamo action to convection in the sun. *C. T. Gordon* reviewed models in which the magnetic fields were maintained by motion in baroclinically unstable flows. A pole–equator pressure difference can give rise to axisymmetric zonal (heliostrophic) flow which is unstable to baroclinic (Rossby) waves, with predominantly horizontal motions. It has been suggested that these baroclinic waves are responsible for the sun's equatorial acceleration (Starr & Gilman 1965), though they are stabilized in the presence of a toroidal magnetic field (Gilman 1967*a*, *b*). This motion also possesses helicity and can serve as a dynamo. Gilman (1969*a*, *b*) computed the motion and fields for a model with Cartesian geometry; Gordon (1972) has extended this treatment to a thin spherical shell. Both models use a truncated spectral representation for horizontal variations while vertical shears are described by introducing two discrete layers. Small seed fields grow by dynamo action and then reverse, with a period that varies from 2 to 12 years. Although the treatment is valid only for a stably stratified layer, the model reproduces the salient features of Babcock's dynamo process.

A more accurate calculation requires a proper understanding of the structure of the sun's convective zone. Recent progress in convection theory was discussed in some detail. *D. O. Gough* described an approach to nonlinear convection theory based on expansion in horizontal eigenmodes of the linearized problem (Gough, Spiegel & Toomre 1973). For Bénard convection in a Boussinesq fluid single mode calculations have been carried out up to a Rayleigh number of 10^{25}. The behaviour of the Nusselt number N for convection between rigid boundaries agrees with the predictions of asymptotic theory ($N \sim (a^2R\ln(a^2R))^{\frac{1}{5}}$, where R is the Rayleigh number and a the horizontal wavenumber in the cell). When two or three modes are included, behaviour is more exotic: time-dependent motion appears, with different scales in the boundary layers and the main flow. *E. A. Spiegel* briefly outlined the results (due mainly to E. Graham and J. Latour) obtained when this technique is applied to a compressible fluid, using the anelastic approximation. The method offers a means of checking the mixing-length theory used in stellar structure. A different approach to nonlinear Bénard convection was described by *D. R. Moore*: he integrated the equations for two-dimensional motions between free boundaries on a computer and extended earlier calculations (Veronis 1966) to a Rayleigh number 1000 times the critical value. The horizontal variations are more complex than those allowed in multi-mode calculations and it was found that for convection between free boundaries the Nusselt number varied as $R^{\frac{1}{3}}$ in the viscous regime but that $N \propto R^{0\cdot 36}$ when advection dominates diffusion in the vertical plumes.† At high Rayleigh numbers persistent nonlinear oscillations appeared in flattened cells (Moore & Weiss 1972).

R. S. Peckover discussed numerical experiments on the effect of convection on magnetic fields in a conducting fluid. He assumed a constant temperature field (i.e. zero Péclet number) but included the dynamical effect of the magnetic field on the motion. Weak fields were swept aside, as expected from kinematic calculations, forming flux ropes between convection cells. When the local magnetic energy density became comparable with the kinetic energy density of the flow

† Gough, Spiegel and Toomre also find $N \sim R^{\frac{1}{3}}$ for free boundaries and high R and sufficiently high Prandtl number.

this process was halted and some form of equipartition was achieved. However, the maximum magnetic energy density can be an order of magnitude greater than the kinetic energy density of convection. The complicated behaviour occurring when rotation is also present was shown by *J. O. Murphy*.† Whereas a magnetic field or rotation applied separately are known to inhibit convection, it is found that the two together can enhance it. Murphy extended the linearized theory (Eltayeb 1972), using the mean field approximation, and showed that the magnetic field can relax the constraint imposed by rapid rotation: for fixed values of R and of a Taylor number sufficient to inhibit convection there is a range of values of the Chandrasekhar number for which convection is possible (though the Nusselt number is always less than it would be in the absence of both rotation and magnetic fields).

F. H. Busse discussed the effects of rotation on convection in a spherical system. For a thin spherical shell, rotating slowly, an expansion about the marginal state shows that the preferred modes are given by sectorial spherical harmonics (P_m^m), corresponding to banana-like cells. The cellular pattern rotates faster than the shell itself and higher order calculations give an equatorial acceleration that increases with depth. This model has been used to explain differential rotation in the sun (Busse 1970*a*). For a rapidly rotating thick shell, convection occurs as rolls in a cylinder parallel to the axis of rotation (Roberts 1958; Busse 1970*b*). This may be relevant to convection in the earth's core and recent experiments show good agreement with the theoretical predictions.

Convective models for solar differential rotation were reviewed by *P. A. Gilman* and related to his recent numerical results for convectively driven differential rotation (Gilman 1973). These calculations are for a model of convection in a Boussinesq fluid uniformly heated from below in an equatorial annulus of a rotating spherical shell, and use the mean field approximation. The problem is solved for Rayleigh numbers up to a few times the critical value and Taylor numbers T in the range $10^2 < T < 10^6$. As T increases, the equatorial regions rotate faster (but with a local equatorial deceleration near the surface). This differential rotation is maintained primarily through momentum transport in the cells, rather than by meridional circulation; indeed the latter is poleward rather than towards the equator at the surface. The cell pattern has a prograde motion and the angular velocity decreases with depth. With a Taylor number of 3×10^4 many features of solar convection are reproduced, and the pattern of motion appears likely to act as a dynamo. These results were compared with the spherical shell models of Busse (1970*a*) and Durney (1970, 1971) and also with models requiring an equatorward circulation (Kippenhahn 1963; Cocke 1967; Kohler 1970; Durney 1972*a*, *b*).

The overall structure of the convective zone appears to be separated into granules near the surface, supergranules deeper down, and giant cells near its base (Bumba 1967; Simon & Weiss 1968). In equatorial regions the giant cells are elongated parallel to the axis of rotation and have an angular velocity corresponding to the 27-day period of the sector structure; but this pattern will not extend to the poles. Magnetic fields are concentrated by the different scales of

† Joint work with R. Van der Borght.

motion (Howard & Stenflo 1972) and dynamo action may be caused by a radial shear of angular velocity in the giant cells, coupled with cyclonic motions in the supergranules (Steenbeck & Krause 1969).

3. Stellar and interstellar magnetism; pulsars

The observational and theoretical background to astrophysical magnetic fields was reviewed by *E. A. Spiegel* and *E. N. Parker*, and the later discussions were concentrated on pulsars and magnetic stars. Spiegel related observed stellar magnetic fields to the evolutionary states of stars. Main sequence stars with surface temperatures less than 8000 °K have deep convective envelopes and are relatively slow rotators. Such stars show calcium emission whose strength correlates with the strength of the magnetic fields, if we may generalize from the solar case. As these cool stars age they lose angular momentum and their magnetic field strengths diminish. These developments are closely related with the extensive hydromagnetic effects of the deep convection zones whose details can be observed on the sun. Here dynamo action is believed to be all-important.

In the stars with surface temperatures higher than 8000 °K, convection near the surface is weak. Certain *A*-type stars with surface temperatures in the range 8000°–15000° show spectral peculiarities and are slow rotators. Magnetic fields have been measured in about 200 of them, with strengths up to 34 kilogauss (Preston 1971; Ledoux & Renson 1966). The observed properties of these stars, including field strengths, usually vary periodically with periods of the order of days.

In the final stages of evolution, stars contract to small radii. For masses below $\sim 1{\cdot}3$ solar masses, dense stars can be supported by the pressure of degenerate electrons. These are white dwarfs whose radii are ~ 6000 km. Magnetic fields have recently been discovered in four of them with strengths $\sim 10^7$ gauss and these do not seem related with surface convection (which does occur in some white dwarfs). Crudely speaking, these fields are stronger than in ordinary stars in the ratio of radius squared.

An even more condensed stellar state is the neutron star which is thought to be formed in the supernova process. The masses of neutron stars cannot be much larger than about 1 M., but the exact value is uncertain. Their radii are $\sim$ 10 km. The pulsars are thought to be rotating neutron stars and their periods range from 30 ms to about 2 sec. Their periods increase with time and in the case of the Crab Pulsar rotational energy is lost at a rate $\sim 10^{39}$ erg/s. If the rotational braking is magnetic, a field strength $\sim 10^{12}$ gauss is indicated, with comparable field strengths derived for all other pulsars whose spin-down can be detected. As with magnetic white dwarfs such field strengths scale roughly like radius squared in comparison with ordinary magnetic stars.

Parker reviewed the general occurrence of magnetic fields and the development of theories of their origin (Parker 1970*a*). Many astrophysical fields are maintained by dynamo action as the combined result of non-uniform rotation and cyclonic motion (i.e. motion with mean helicity, see § 6 below) which occurs naturally in rotating convecting bodies. The essential physical properties of field generation can be shown by assuming cylindrical polar co-ordinates (ϖ, ϕ, z). Then non-uniform rotation $\mathbf{v}_\phi(\varpi, z)$ generates a strong large-scale toroidal field $\mathbf{B}_\phi$, according to

$$\frac{\partial \mathbf{B}_\phi}{\partial t} - \eta \nabla^2 \mathbf{B}_\phi = \mathbf{e}_\phi \left(\mathbf{B} . \nabla \left(\frac{v_\phi}{\varpi} \right) \right),$$

in the axisymmetric case, where $\mathbf{e}_\phi$ is a unit vector in the ϕ direction and η is the magnetic diffusivity. Similarly, for the toroidal vector potential $\mathbf{A}_\phi$, neglecting mean motion,

$$\frac{\partial \mathbf{A}_\phi}{\partial t} - \eta \nabla^2 \mathbf{A}_\phi = \Gamma \mathbf{B}_\phi,$$

where Γ is a measure of the mean cyclonic velocity component (Parker 1955, 1970) which may be identified with the α of Steenbeck & Krause (1969). The general behaviour of solutions to these equations is easily studied in Cartesian geometry and depends on the product $\Gamma d(v_\phi/\varpi)/d\varpi$. The behaviour of the earth's field depends critically on the distribution of cyclones (Levy 1972*a*, *b*). The outstanding question for the sun is the form of the non-uniform rotation: dynamo waves migrating towards the equator require that angular velocity should decrease with radius.

The galactic magnetic field can be measured by the Zeeman effect and Faraday rotation. It has a magnitude of $3\text{–}4 \times 10^{-6}$ gauss but no unique description emerges from the observations. Once again, a dynamo model can be based on a combination of non-uniform differential rotation and local cyclonic turbulence (Parker 1971*a*, *b*).

L. Mestel discussed the interaction between rotation and magnetic fields in magnetic stars. The oblique rotator model for magnetic variable stars had been referred to by Spiegel, who noted in particular Preston's conclusion (1971) that the angle of obliquity χ between the magnetic and rotation axes tends to be near either 0 or $\frac{1}{2}\pi$. Mestel argued that the lack of correlation between spectral type, rotation, and surface magnetic field suggests that the fields of these stars are not maintained by dynamo action (Krause 1971) but rather are slowly decaying fossil fields (Cowling 1945). A large-scale stellar field is probably dynamically stable only if toroidal flux loops link the (observable) poloidal field (Wright 1970, 1973). Dissipation of the energy of internal motions causes the star to rotate about its axis of maximum moment, yielding high or low obliquity according as the star is dynamically prolate (toroidal flux dominant) or oblate (poloidal flux dominant) (Spitzer 1958; Mestel & Takhar 1972). Stars will be visibly magnetic as long as flux is not dragged below the surface by rotationally driven circulation (Mestel 1965, 1971*b*, 1972; Wright 1969); thus the magnetic stars should be mainly slow rotators, as is observed (Preston 1970). Angular momentum may have been lost through magnetic coupling with a wind emitted during the early convective phase (Mestel 1968*a*), a process we see occurring in the sun with its dynamo-built surface field. (Such coupling also affects the angle χ (Mestel 1968*b*; Mestel & Selley 1970; Selley 1973), though less powerfully than the dissipation process.) More probably, loss of angular momentum occurs by direct coupling with the interstellar gas, a process better described by fluid equations (Chia & Henriksen 1972) than by a single-particle analysis (Havnes & Conti 1971). These ideas also have their counterparts in theories of star formation and of the dynamics of white dwarfs and pulsars (Mestel 1971*a*).

A. Hewish introduced the discussion of pulsars with an account of recent observations. Successive pulses differ from each other but the mean pulse shape does not vary very systematically with period. The beam might be expected to become narrower as the period and the radius of the light cone increase and there is some observational evidence for this. A pulse envelope is frequency dependent, particularly when observed at frequencies below 100 MHz. Individual pulses are often highly polarized (usually elliptically) but the average linear polarization shifts continuously and systematically through the envelope. A curious feature is the existence of drifting subpulses, which overtake the main pulse at drift rates that vary by an order of magnitude, though the gap between successive subpulses remains roughly constant (Hewish 1970).

The discovery of neutron stars in 1968 provided a physicist's paradise, which was described by *V. Canuto*. With an average density of 10^{14} g cm^{-3}, the average separation of particles is only 10^{-13} cm, while the Compton wavelength is only one-fortieth of the Larmor radius. The outer solid crust, about 500 m deep, is a lattice of nuclei, with a conductivity a million times greater than that of copper. The neutron liquid (probably a super-fluid) surrounds a solid core of neutrons and charged hyperons, about 2 km in radius (Ruderman 1969, 1972; Cameron 1970). The opacity in the surface layers is altered by the strong magnetic field so that neutron stars cool more quickly than had previously been supposed. It is thought that sudden small changes in the rotation rates of pulsars correspond to 'star-quakes' in either the solid crust or core. *J. Bazer* discussed the problem of wave propagation in a magnetoelastic medium. A geometrical theory (analogous to geometrical optics) has been developed for the linearized equations (Bazer 1972); nonlinear simple waves have also been investigated (Bazer & Karal 1972) and progress has been made in studying the development of shocks.

J. E. Gunn reviewed electromagnetic pulsar models (Ostriker & Gunn 1969). The region outside the neutron star is almost a vacuum but surface electric fields can accelerate particles outwards along the lines of force to produce a static space charge throughout the magnetosphere. The energy loss resulting from spin-down can be calculated from the magnetic torque at the light cylinder and is sufficient to account for the radiated energy. Outside the light cylinder the field depends on the inclination of the dipole axis. If the magnetic and rotational axes are perpendicular, waves are emitted and these can accelerate particles to energies of 10^{14} V. *S. C. Michel* reported a lack of consensus on radiative mechanisms, though the overall picture was generally accepted. With an aligned dipole, corotating plasma would extend to a shock near the light cylinder, while a stellar wind streamed out from regions near the pole and a high density plasma would be formed by shock heating in the equatorial plane.

The Session concluded with a comprehensive review by *B. Lehnert* of theoretical and experimental investigations on rotating plasmas (Lehnert 1971), and a short talk by *V. G. Endean* who suggested that lack of success in attempts to verify experimentally Ferraro's law of isorotation could be explained only through recourse to a fundamental modification of Ohm's law in a rotating conducting fluid; in the discussion that followed, few were prepared to accept this conclusion, but it was agreed that there was as yet no convincing experimental verification of Ferraro's law, largely because laboratory conditions do not approximate to the conditions in which the law may be expected to be valid.

4. Boundary layers, detached shear layers, and spin-up problems

The hydromagnetic spin-up problem is interesting from the purely magnetohydrodynamical viewpoint because it represents such a dramatic example of how the Lorentz force can affect the distribution of fluid angular momentum when that force is rotational; thus, the entire bulk of an electrically conducting fluid can be made to spin faster as a result of a Lorentz body torque induced by shearing motions that arise in boundary layers. Such flows require, for their understanding, a knowledge of viscous and resistive boundary layers, detached shear layers, and multiple wave systems (e.g. Alfvén waves and inertial oscillations). In addition, physical motivation for MHD spin-up is provided by the various cosmical bodies with strong magnetic fields that appear to be undergoing changes in angular velocity. This session was mainly concerned with situations of astrophysical interest, although some work related to the earth's core was also reported.

E. A. Spiegel began by reviewing the astronomical motivation for studying spin-down (here the phrase is used in the broad sense to denote any process, whatever the time scale, by which the angular velocity of a rotating fluid body changes). Observations of solar type stars (Kraft 1967) suggest that their rotational velocities decay algebraically with age, perhaps as $t^{-\frac{1}{2}}$, sufficiently late in their evolution (Skumanich 1972). Since different stars presumably had different initial conditions, it need not follow that any one star follows such a decay law. However, should this be the case, then nonlinear processes (such as the dependence of solar wind torque on angular velocity, Durney & Stenflo 1972) clearly contribute to the rotational braking, because linear spin-down proceeds exponentially in time (Greenspan & Howard 1963).

Other cosmical bodies thought to be spinning down include the peculiar *A* stars and pulsars. Since pulsars are thought to be rotating neutron stars with solid inner core and outer crust, enclosing a superconducting liquid, spin-down could involve quantized vorticity, a novel situation.

The electromagnetic torque exerted by the solar wind (Weber & Davis 1967) slows the tenuous convective envelope of the sun, but the nearly steady observed surface angular velocity implies that a flux of angular momentum is supplied from below; consequently, the radiative core must be spinning down, and the central issue in solar spin-down is one of time-scale. Lumped parameter models of this process suggest that the half-life of core angular momentum is about the same as the present age of the sun (which may not be coincidental). Dicke's contribution to this question and the oblateness measurement (Dicke 1970) were also reviewed by Spiegel.

A major complicating feature of the sun is the highly stable density stratification beneath the convection zone. Spin-down in a Boussinesq fluid is, therefore, a relevant idealized problem, and the comparatively large existing literature for non-magnetic linear flow in a cylinder (Clark 1972) was reviewed by *A. Clark*, with emphasis on the influence of buoyancy on (i) Ekman layers, (ii) sidewall layers, and (iii) interior circulations. On the time-scale for homogeneous Ekman spin-down, only a thin layer of interior fluid spins down (when the stratification is strong), so a quasi-steady state of non-uniform rotation is realized. A similar result occurs in a sphere with central gravitational field (Clark, Clark, Thomas & Lee 1971). Should the Prandtl number be small (as it is deep in the sun, because of efficient radiative conduction), complete spin-down occurs only on the long time-scale for Eddington–Sweet circulations (Sakurai, Clark & Clark 1971).

E. R. Benton reviewed the theory of hydromagnetic spin-up for a homogeneous electrically conducting fluid contained in an insulating circular cylinder with an initially uniform axial applied magnetic field $\mathbf{B}_0$. Finite amplitude changes in angular speed were allowed, but the effects of sidewall boundary layers ignored (Benton 1973). With $\mathbf{B}_0$ first set to zero, the strongly nonlinear hydrodynamic cases of spin-up from a state of rest (Wedemeyer 1964) and spin-down to a state of rest were discussed and illustrated by a film. Spin-up from rest is distinguished by a propagating detached shear layer which separates a non-rotating cylindrical core from fluid forced to rotate by viscous action at sidewalls and end-plates. It was noted that, in spin-down to rest, sidewall boundary layers are prone to Rayleigh centrifugal instability, and Ekman layers may also be unstable; a resultant emission of inertial oscillations into the interior could mix the angular momentum and hasten the spin-down process (compared to theoretical predictions, which ignore instabilities).

The moderate Rossby number nonlinear theory of Greenspan & Weinbaum (1965) was discussed and a failure of that theory for strongly nonlinear spin-up was noted. An alternative method (Benton 1973), providing uniform validity in Rossby number, was reported; it predicts that inertial effects will cause both spin-up and spin-down to lengthen (compared with linear theory). Strongly nonlinear spin-down is found to decay toward the asymptote algebraically in time rather than exponentially, which may be suggestive for stellar spin-down.

In hydromagnetic spin-down, Ekman layers are replaced by Ekman–Hartmann layers. Linear theory (Gilman & Benton 1968; Benton & Loper 1969; Loper & Benton 1970) shows how an induced Hartmann current system overcompensates for an attendant suppression of Ekman pumping to produce more rapid spin-down than in linear non-magnetic spin-down (Greenspan & Howard 1963). Recent work (Benton & Chow 1972; Benton 1973) on the nonlinear version of this problem was summarized. Inertial nonlinearity always retards both spin-up and spin-down, but the magnetic coupling acts in the opposite sense. However, for strongly nonlinear flow, a weak field is very ineffective in spin-down (because it is continuously dispersed laterally by the secondary flow) but highly effective in spin-up (because of field line stretching by secondary flow). Strong magnetic effects (in the sense |Lorentz force| $\gg$ |Coriolis force|) dominate all nonlinear processes and give rapid spin-up times identical to those for spin-down.

J. C. R. Hunt described a wide variety of sidewall layers and detached shear layers that arise in mechanically and electrically driven MHD channel flows at large Hartmann number (see Hunt & Shercliff 1971, for a review). The strong analogies (like Taylor–Proudman constraints, and control of core flows by Hartmann layers) with non-conducting but rotating fluids was made especially evident.

A group at Florida State University is developing laboratory experiments on rotating magnetohydrodynamics with liquid metals, where careful measurements of flow speed and electric field are planned, using heated thermistor beads and an ultrasonic Doppler velocimeter. *W. W. Fowlis* described the apparatus and pointed out that both steady motions driven by differential rotation of top and bottom lids and unsteady spin-up flows are contemplated. A long-term aim is the construction of a laboratory hydromagnetic dynamo, a project that deserves every encouragement.

D. Loper described the somewhat elusive transient magnetic diffusion region (Benton & Loper 1969; Loper & Benton 1970; Loper 1973) and then turned to the effect of boundary conductivity on linear hydromagnetic spin-up (Loper 1970*a*, *b*; 1971). Here an extra loop of electric current flows into the boundaries (possibly as in the earth's mantle) and further couples together the interior fluid and the boundary. In this context, the strength of coupling is not simply the ratio of boundary conductivity to that of the fluid but rather the ratio of boundary *conductance* to that of the fluid contained in one Ekman depth (this is large for the earth!).

A laboratory experiment by A. McEwan to demonstrate angular momentum mixing by turbulence was described briefly by *J. J. Monaghan.* Fluid contained in a circular cylinder fitted with a stretched rubber membrane for a lid is set in uniform rotation and then the lid is oscillated vertically at a preselected frequency to produce inertial waves which are thought to degenerate resonantly into turbulence. The subsequent visualization of intense vortices is associated with the mixing of angular momentum.

Since the linear spin-up of a stratified non-conducting fluid and a homogeneous electrically conducting fluid are now quite well understood, a useful next step is probably to add stratification and hydromagnetic coupling together. However, before attempting that, it is necessary to understand rotating hydromagnetic sidewall boundary layers (somewhat as in Ingham 1969, but without the restriction to small magnetic Reynolds number). Moreover, the possibilities of topographic features on the core–mantle interface (see Session A) suggests the necessity for studying spin-up in unconventional containers. Then the crucial but difficult question of stability of these flows must be confronted.

5. Waves and instabilities subject to Coriolis and/or Lorentz forces

Hydromagnetic planetary waves (Hide 1966; Malkus 1967; Stewartson 1967) and their counterpart in unbounded systems, namely hydromagnetic inertial waves (Lehnert 1954), are characterized by magnetostrophic balance between Coriolis and Lorentz torques acting on individual fluid elements. They have periods $\tau \sim L^2\Omega/V^2$ (where L is a typical length, Ω the angular speed of basic rotation and V the Alfvén speed), which are typically very much longer than those of ordinary Alfvén waves, L/V, and of ordinary planetary or inertial waves, $\gtrsim \Omega^{-1}$. The quantity τ is $\lesssim 10^9$ s for the sun (Lehnert 1954; Gilman 1968), $\lesssim 10^9$ s for the planet Jupiter, $\gtrsim 10^4$ s for the Crab Pulsar (Hide 1971*a*), and if the strength of the toroidal magnetic field in the earth's liquid core is ~ 100 gauss then $V \sim 10\,\mathrm{cm\,s^{-1}}$ and $\tau \sim 10^{10}$ s (300 years), which is comparable with the time-scale of the geomagnetic secular variation. Hydromagnetic planetary waves in a thin spherical fluid shell propagate eastward (Hide 1966; Stewartson 1967), but in a thick shell, such as the earth's liquid core, both eastward and westward propagating waves are possible in principle (Hide 1966; Malkus 1967; Hide & Stewartson 1972) unless the vertical temperature gradient is so highly sub-adiabatic as to inhibit westward-propagating modes (by preventing vertical motions).

The idea that the westward drift of the non-dipole part of the geomagnetic field relative to the equatorial dipole could be a manifestation of westward-propagating hydromagnetic planetary waves in the core (Hide 1966) gains support from recent theoretical studies, which include investigations of (*a*) the relationship of hydromagnetic planetary waves to eigenoscillations of a finite body of fluid, especially thin and thick spherical shells (Malkus 1967; Stewartson 1967; Stewartson & Rickard 1969; Hide & Stewartson 1972); (*b*) effects due to stable density stratification (Hide 1969, 1971*b*; Kalra 1969; Acheson & Hide 1972); (*c*) propagation and critical layer absorption of hydromagnetic inertial waves in a non-uniform magnetic field, including so-called 'valve' effects (Acheson 1972*a*, 1973*a*; McKenzie 1973; Rudraiah & Venkatachalappa 1972); (*d*) wave-generation by various types of three-dimensional instability (Acheson 1972*b*, 1973*b*; Booker 1972; see also Braginskii 1967; Roberts & Soward 1972); and (*e*) attenuation mechanisms (Acheson & Hide 1973).

Some of the main results of these studies were outlined in *R. Hide's* introductory remarks, which also included a discussion of the governing equations and basic approximations. In the first of the papers that followed *K. Stewartson* considered some of the theoretical difficulties encountered in the analysis of eigenoscillations of thick spherical shells of fluid, pointing out that future progress will require basic mathematical studies of hyperbolic partial differential equations under boundary conditions of the Dirichlet–Neumann type and emphasizing the need to establish from first principles the extent to which concepts such as group velocity are valid when dealing with oscillations governed by such equations and boundary conditions. *J. F. McKenzie* and *D. J. Acheson* then dealt with various aspects of the propagation of hydromagnetic waves in a rotating fluid when the basic magnetic field is non-uniform, including critical layer absorption. Acheson also discussed the stability of an annular system with radial gradients of basic zonal motion, magnetic field strength and buoyancy, showing that instabilities tend to manifest themselves as *westward* propagating hydromagnetic inertial waves. In the final paper *J. R. Booker* considered the stability of magnetostrophic flows in a sphere. The stability criteria are remarkably similar to those found in the annular case. Moreover, non-axisymmetric instabilities, when they occur, show the same preference for westward propagation as is found in annular systems.

6. Dynamo theory

Dynamo theory was the central theme running through the proceedings. The stage was set early by Bullard, and by Parker, who reviewed the occurrence and character of cosmical fields. It was stressed that, at least in some instances (certainly in the case of the earth, and debatably in the case of the sun), the free decay time, $\tau_\eta = \mathscr{L}^2/\eta$, of the large-scale current system is so small that a generation mechanism must be sought. This, of course, provides the motivation for dynamo theory. (Here $\mathscr{L}$ is a typical dimension of the body, and η is a magnetic diffusivity which requires further interpretation – see below. When molecular conduction alone operates, $\eta = 1/\mu\sigma$, where μ is permeability and σ is electrical conductivity.)

The mathematical nature of the problem was reviewed by *P. H. Roberts*. The simplest theoretical case is provided by the kinematic dynamo problem, in which all dynamical questions are ignored, and the electrodynamics alone is studied. The velocity $\mathbf{u}$ of the conducting fluid is given. (Here, as usual, we take $\operatorname{div} \mathbf{u} = 0$.) The question of regeneration poses a linear (vector) eigenvalue problem for β, the growth rate of the field $\mathbf{B}$ (see, for example, Roberts 1971*b*). Attention is focused on the growth rate with largest real part, since this determines the long term behaviour of $\mathbf{B}$. This β may be real (d.c.) or complex (a.c.). Conditions are sought under which $\mathbf{B}$ is self-excited, i.e. under which the largest $\mathscr{R}(\beta)$ is positive.

The dynamo will fail if the assumed $\mathbf{B}$ is axisymmetric (Cowling 1933), if the fluid conductor V is spherical and the assumed $\mathbf{u}$ is nowhere radial (Elsasser 1946), if the rate of strain in the flow is 'too slight' (Backus 1958), *or* if the fluid velocity in V is everywhere 'too small' (Childress 1969). A convenient dimensionless measure of the fluid velocity is the magnetic Reynolds number $R = \mathscr{U}\mathscr{L}/\eta$, where $\mathscr{U}$ is a typical magnitude of $\mathbf{u}$. The first two of the antidynamo theorems just stated imply some complexity for the field and/or flow postulated; the last two rule out all ideas of solution by expansion about $R = 0$. Taken together they account for much of the tortuous development of the subject. For example direct numerical integrations of the dynamo equations are necessarily intricate. Moreover, it is found that, purely as a result of too crude a truncation, solutions can be obtained in situations in which it is known (by the first or second antidynamo theorem) that none exists.

I. Lerche described to the meeting one numerical procedure based on the variational formulation of the dynamo equations (Gibson & Roberts 1967). The variational problem is not self-adjoint, and the true eigenvalue is therefore neither an upper nor a lower bound for those obtained by approximate trial functions.

H. Köhler propounded a novel grid-point method with which he plans to solve the dynamo equations iteratively (see also §2 above). The continuity of $\mathbf{B}$ across the surface S of V gives rise to a non-local boundary condition, i.e. one which connects the field at each point of S to all other points on S. Köhler's treatment of this condition was not unrelated to one which *R. Thirlby* later described to the meeting, and which led to good theoretical results in test cases.

The difficulty of the non-local boundary condition is entirely avoided by the method of Bullard & Gellman (1954). The field in V (assumed spherical with centre O) is first split into toroidal and poloidal parts, $\operatorname{curl} T\mathbf{r}$ and $\operatorname{curl}^2 S\mathbf{r}$ respectively, where $\mathbf{r}$ is the radius vector from O, and $\mathbf{u}$ is written similarly. The defining scalar T (and likewise S) is further divided into linear combinations of the surface harmonics, $P_n^m(\theta)\sin m\phi$ and $P_n^m(\theta)\cos m\phi$, where (r, θ, ϕ) are spherical polar co-ordinates. The corresponding coefficients T_n^{ms} and T_n^{mc} (and likewise S_n^{ms} and S_n^{mc}) are functions of r alone and are therefore governed by coupled ordinary differential equations.

Bullard & Gellman proposed a simple dynamo consisting of a shear T_1 and an upwelling S_2^{2c} but did not obtain convergence; Gibson & Roberts (1969), and Lilley (1970), took the calculation further, but without success. Lilley also examined a combination of T_1, S_2^{2s} and S_2^{2c} flows, and obtained apparent convergence. P. H. Roberts (1972) took the calculation further, however, and showed that Lilley's dynamo also failed, a conclusion supported at the meeting by *D. Gubbins*. P. H. Roberts reported apparent convergence for a flow synthesized from the T_1, S_2, S_2^{2s} and T_3^{2s} harmonics, and integrated by S. Kumar and himself, using the parallel shooting method. Gubbins, following G. O. Roberts (reported by P. H. Roberts 1971*a*), selected an axisymmetric motion consisting of many (n) cells. Solutions proportional to $\exp(im\phi)$ exist, for which the dynamo equations involve only two space dimensions (r and θ). Gubbins displayed convincingly converged results. He concluded that the critical R, based on the maximum flow speed, decreases as n increases, and tends to a non-zero limit. As Bullard observed, this lends added support to the two-scale dynamos described below. Gubbins also reported that the most efficient dynamos were those in which the magnetic Reynolds number based on the largest *poloidal* velocity in V was about unity.

The difficulties of direct computation suggested that alternatives should be considered. Asymptotic methods have been used with great success and, in fact, provided both of the original existence proofs of the subject (Herzenberg 1958; Backus 1958). The best known is the two-scale method, in which it is supposed that the flow and magnetic field exist on two widely disparate length-scales L ($\ll \bar{L}$) and $\bar{L}$ ($\approx \mathscr{L}$), called the microscale and the macroscale, respectively. The existence of dynamo action has been rigorously demonstrated by Childress (1967, 1970) and G. O. Roberts (1970, 1972). Briefly the microscale flow $\mathbf{u}'$ induces from the macroscale field $\bar{\mathbf{B}}$ a field $\mathbf{B}' \approx O(R_m \bar{B})$, varying on the microscale (assuming that $R_m = u'L/\eta$, the microscale Reynolds number, is small). The induced e.m.f. $\mathbf{u}' \times \mathbf{B}'$ created by both microscale fields possesses a macroscale component, $\mathscr{E} = \overline{\mathbf{u}' \times \mathbf{B}'}$, of order $R_m^2 B/L$, which can act to reinforce $\bar{\mathbf{B}}$ itself. But to counter macroscopic dissipation, it must be of order $\eta \operatorname{curl} \bar{\mathbf{B}} = O(\eta \bar{B}/\bar{L})$, or larger. Thus $R_m^2 \bar{L}/L$ should be $O(1)$ or greater. In fact, Childress's central limiting process was $R_m \to 0$, $\bar{L}/L \to \infty$, with $R_m^2 \bar{L}/L$ held fixed.

Closely related ideas apply to fields generated by turbulent flows, for which $\bar{\mathbf{B}}$ is defined as the average over an ensemble of identical systems, and $\mathbf{B}' = \mathbf{B} - \bar{\mathbf{B}}$ is then the fluctuating remnant. Since Maxwell's equations are linear, they also govern $\bar{\mathbf{B}}$. The electromotive force $\mathscr{E}$ set up by the microscale gives rise to a new form for Ohm's law for $\bar{\mathbf{B}}$; the resulting subject has been named 'mean field electrodynamics' by F. Krause, K. H. Rädler and M. Steenbeck.† It is easy to see that $\mathscr{E}$ is a linear functional of $\overline{\mathbf{B}}$, but to estimate it a closure approximation is necessary. Commonly 'first order smoothing' is adopted, i.e. the fluctuating e.m.f.,

$$\mathbf{u}' \times \mathbf{B}' - \overline{\mathbf{u}' \times \mathbf{B}'},$$

is discarded, a procedure which should be acceptable for small R_m or small $u'T/L$, where T is the correlation time.

K. H. Rädler, speaking for Krause and himself, surveyed the achievements and present status of mean field electrodynamics. He first asked whether incompressible statistically-steady isotropic mirror-symmetric turbulence could regenerate field. He concluded in the negative, and gave an argument by Krause & Roberts (1973) based on Bochner's theorem. (Further independent corroboration was provided in a special case by *J. Gilliland* & *K. Aldridge*.) He pointed out further that (in a first approximation) $\mathscr{E} = -\beta \operatorname{curl} \mathbf{B}$, where β is a constant determined in principle by the statistical properties of the turbulence, so that the diffusivity of the mean field is essentially $\eta_T = \eta + \beta$. Since $\beta > 0$ (Krause & Roberts 1973) it follows that the turbulent conductivity, $\sigma_T = 1/\mu\eta_T$, is less than the molecular. Rädler also stressed the analogy between the evolution of mean field in a turbulent electrically conducting fluid, and the behaviour of the mean temperature in a turbulent thermally conducting fluid. In fact, in first-order smoothing theory, both are governed by the same dispersion relationship (Krause 1972), and the application of Bochner's theorem by Krause & Roberts (1973) showed that both decayed with time. Lerche conceded at the meeting that some of his publications (Lerche 1971*a*, *b*; Lerche & Low 1971) which purported to prove otherwise must be in error.

Next Rädler considered whether pseudo-isotropic turbulence (which is isotropic but not mirror-symmetric) could be more successful. He concluded that it could, and reminded his audience of the discovery of the α-effect, in which $\mathscr{E} = \alpha\overline{\mathbf{B}}$ (see Steenbeck, Krause & Rädler 1966). Once this term is included in the mean-field induction equation, the antidynamo theorems are seen not to apply to $\overline{\mathbf{B}}$, and comparatively simple (e.g. axisymmetric) solutions may be found. Although the α-effect is attractive in its simplicity, there is, as Rädler emphasized, no practical method known of realizing pseudo-isotropy, and non-homogeneous flows have to be considered. For example, the α-effect should arise in a body rotating with angular velocity $\boldsymbol{\omega}$ in the presence of density stratifications or a gradient $\mathbf{g}$ in turbulent intensity. Then, however, other terms proportional to $(\mathbf{g}.\overline{\mathbf{B}})\boldsymbol{\omega}$ and $(\boldsymbol{\omega}.\overline{\mathbf{B}})\mathbf{g}$ also arise in $\mathscr{E}$, and these might conceivably act against α-generation. Rädler reported preliminary results showing that, when included with the α-effect, dynamo action becomes easier rather than more difficult. (α itself would be proportional to $\boldsymbol{\omega}.\mathbf{g}$ and would therefore be of opposite sign in opposite hemispheres.) Rädler also called attention to the existence of special mean electromotive forces, not of α-effect type, which arise from homogeneous turbulence, axisymmetric with respect to (say) a rotation axis. These might also be capable of sustaining field.

† Most of the writings by these authors appeared originally in German; they have recently been translated into English (Roberts & Stix 1971).

Many models of working axisymmetric dynamos incorporating the α-effect have now been constructed. These fall into three main types: the α^2-dynamos, the $\alpha\omega$-dynamos, and the $\alpha\omega$-dynamos with meridional circulation. First, regeneration of mean field is possible through the action of turbulence alone without the aid of bulk motions. Meridional poloidal (P) field arises from azimuthal toroidal (T) currents, which are generated from T fields by the α-effect; similarly, T field arises from P currents created from P fields by the α-effect. Functioning therefore on a product of two α-processes, the name 'α^2-dynamo' is appropriate. Numerical computations indicate that these dynamos are d.c., and efficient, as judged by the α-effect Reynolds number $R_\alpha = \tilde{\alpha}\mathscr{L}/\eta_T$, where $\tilde{\alpha}$ is a tpyical magnitude of α.

In many astrophysical circumstances, the creation of T field from P field through the α-effect is swamped by its production by a large-scale toroidal shearing motion, and may be neglected. This is the case when R_α is small compared with the shear Reynolds number $R_\omega = \tilde{\omega}'\mathscr{L}^3/\eta_T$, where $\tilde{\omega}'$ is a typical gradient of shear. The α-effect must still be retained for the creation of P field from T. The resulting picture is much as Parker (1955) originally presented it, although much more information is now available about the behaviour of models, and this was surveyed at the meeting. P. H. Roberts proposed as a general, but not universal, rule that when $\alpha\omega' < 0$ throughout the northern hemisphere (with respect to the direction of $\boldsymbol{\omega}$), the easiest dynamo to excite is of dipolar parity and is a.c., the patterns of magnetic activity drifting from poles towards the equator, as for Maunder's butterfly diagram for the sun. If, on the other hand, $\alpha\omega' > 0$ in the northern hemisphere, the preferred dynamo is of quadrupole parity and is a.c., the patterns moving polewards from the equator. Some field cycles and butterfly diagrams were presented both by *F. Krause* and by *P. H. Roberts* which supported these statements. The sense of migration is that anticipated from the plane dynamo wave of Parker (1955). Parker described to the meeting how the wave propagated; *M. Stix* presented a generalization which allowed in a crude way for the decrease of α with B expected from dynamical considerations (see below and also Stix 1972).

The idea that $\alpha\omega$-dynamos might be expected to be a.c. as a rule was disputed by Parker, who felt that the δ-function model of Levy (1972*a*, *b*) could be used to prove otherwise. In support, Stix reported an unpublished application of the Gibson–Roberts variational method to that model. He found that, if $\alpha\omega' > 0$, the d.c. dynamo modes found by Levy were indeed preferred over the a.c.; for $\alpha\omega' < 0$, however, he discovered that a.c. dynamo action occurred at smaller Reynolds number, unless the sources of α-effect were placed in low latitudes. Parker also cited his layer model of the galactic dynamo (e.g. Parker 1971*a*) in which a d.c. mode had the lowest dynamo number $R_\alpha R_\omega$, i.e. could be the most easily excited. Late in the meeting, *S. Childress* and *A. M. Soward* discussed yet another layer model in which a.c. dynamo action was favoured.

To add further complications, it can happen (as Rädler noted) that the mode of $\alpha\omega$-dynamo action easiest to excite is asymmetric, and is then necessarily time dependent, the field drifting round the symmetry axis of α and ω. The situation remains unclear, but evidently deserves further attention.

The addition of meridional circulation m has a profound effect on the $\alpha\omega$-dynamos (Braginskii 1964*c*; P.H. Roberts 1972). If m is sufficiently large, the preferred dynamo appears to be d.c. If it is in the right sense, the efficiency (as measured by the critical dynamo number) may be considerably enhanced. P. H. Roberts reported that it seemed to be generally, but not universally, true that $\alpha\omega' > 0$ (< 0) in the northern hemisphere favoured dynamos of dipolar (quadrupolar) parity. He suggested that meridional circulations might be required to account for the general steadiness of the geomagnetic field. *D. E. Loper* remarked that strong meridional circulations might be expected from the non-linear boundary layer described in his talk.

Asymmetric fields can be excited by the $\alpha\omega$-mechanism about as readily as symmetric fields (Krause 1971; Stix 1971; Roberts & Stix 1972). *F. Krause*, who surveyed the models, suggested that they provided a viable explanation of magnetic stars. If so, he observed, the generally accepted picture of an oblique rotator would have to be restricted to fields showing reflexion symmetry with respect to the equatorial plane (see § 3).

Little was said in the meeting about the other well-known asymptotic approach to the dynamo problem, the small asymmetry limit of Braginskii (1964*a*, *b*). By Cowling's theorem, a completely symmetric $\mathbf{B}$ cannot be maintained. If, then, a nearly symmetric field and motion is selected, the marginal R for regeneration should approach infinity as the degree of asymmetry is reduced to zero. This limit, $R \to \infty$, allows theoreticians to avail themselves of asymptotic methods; moreover, many cosmical fields show a high degree of axisymmetry, so that the resulting theory is not an obvious museum piece. As developed by Braginskii, the theory led to dynamos of $\alpha\omega$-type, with or without meridional circulation.

A. M. Soward presented a more general form of the $R \to \infty$ limit, and showed how α^2-dynamos could arise, as well as those based on the $\boldsymbol{\omega} \wedge \mathbf{j}$ effect of Rädler (1969). Soward's technique (1972) rests on a mixed Lagrangian–Eulerian framework, and is a generalization of the method of Frieman & Rotenberg (1960). It was pointed out by *H. K. Moffatt* that the approach could not be used for velocity fields with linked streamlines.

The linearity of the kinematic dynamo problem, although providing simplification, introduces a physical absurdity: if the magnetic Reynolds number exceeds (even only slightly) the critical value for marginal regeneration, the field will grow indefinitely. In practice, of course, the growth would be halted by the Lorentz force, which would modify and reduce the flow (Lenz's law) until amplification is halted. To describe this self-adjustment mechanism the dynamical equations must be restored. Further, in order to support the Joule expenses of the field, an energy source must be postulated. This may be chosen to simplify the theory (as in the models of Moffatt, Thirlby and G. O. Roberts described below), or it may be given by a physical reason, such as thermal buoyancy (as in the models of Busse, Childress and Soward described below). The resulting theory defines the magnetohydrodynamic (MHD) dynamo problem.

We have noted that the α-effect is associated with non-mirror-symmetric motions. As Parker (1955) recognized, a simple class of such flows consists of right-handed or (left-handed) screw motions in which the velocity, $\mathbf{u}'$, and vorticity $\boldsymbol{\omega}' = \text{curl}\,\mathbf{u}'$ are correlated (or anticorrelated), i.e. $\overline{\mathbf{u}'.\boldsymbol{\omega}'} > 0$ (or $\overline{\mathbf{u}'.\boldsymbol{\omega}'} < 0$). On Alfvén's frozen field picture, such a motion will create a loop in a line of large-scale field $\overline{\mathbf{B}}$, and the electric current, $\mathbf{j}' = \mu^{-1}\,\text{curl}\,\mathbf{B}'$, associated with the corresponding change $\mathbf{B}'$ in field will be anti-correlated (or correlated), i.e. $\mathbf{B}'.\mathbf{j}' < 0$, leading to negative α (or $\mathbf{B}'.\mathbf{j}' > 0$, leading to positive α). Clearly, for given driving force, the larger $\overline{\mathbf{B}}$, the less the motions will be able to bend the field lines, and the smaller α will be. Thus, although α is independent of $\overline{\mathbf{B}}$ for the kinematic dynamo problem, it is a decreasing function of $\overline{\mathbf{B}}^2$ in the MHD dynamo. This provides a regulation mechanism for regeneration. It was incorporated qualitatively in a plane layer model which *M. Stix* discussed at the meeting (see also Braginskii 1970; Stix 1972). A different kind of field limiting mechanism, namely the depletion of toroidal flux through sunspot eruption, was included in the model of the solar cycle by Leighton (1969), which Weiss reviewed.

A quantitative model of $\overline{\mathbf{B}}$-dependent α has been studied by *H. K. Moffatt* (1970*b*, 1972) and was described by him during the meeting. Moffatt first considered general properties of the quantity $\mathbf{u}.\boldsymbol{\omega}$, 'the helicity' of the flow. In the absence of field and with kinematic viscosity ν set zero, it is (when integrated over a fixed container) conserved during the motion; indeed, it is a topological invariant related to the winding of vortex tubes about each other (Moffatt 1968). Moffatt showed that inertial waves in a rotating flow possess helicity; he observed that (taking $\boldsymbol{\Omega}$ upwards) rising waves have negative helicity, and falling have positive helicity. This suggests that a random superposition of (say) upwardly propagating waves will be associated with dynamo action. The kinematic theory (Moffatt 1970*a*) shows that the α-effect is not isotropic, but $\mathscr{E}$ is nevertheless proportional to the components of $\overline{\mathbf{B}}$ (i.e. $\mathscr{E}_i = A_{ij}B_j$). The wavelength l of the mode with the greatest growth rate was located, and the way in which A_{ij} varied as $\overline{\mathbf{B}}$ increased through the dynamo action was studied. It was particularly noted that, since the field was at all times force-free, the basic assumption of the theory (that $\hat{\mathbf{u}} = 0$ in the rotating frame) was not violated. If L and T now refer to the correlation length and time of the body force exciting the waves, he found that the ratio of magnetic to kinetic energy is of order $(\nu\Omega T/\eta)^{\frac{1}{2}}\,(l/L)$, at least in conditions which might be expected to apply to the earth's core.

Moffatt emphasized that one of the big uncertainties of the theory is the mechanism by which, in the cosmical context, upward propagating inertial waves (say) are preferentially selected, for if there is equal representation of upward and downgoing waves the dynamo will fail. A similar difficulty is encountered in the other main approach to the magnetohydrodynamic dynamo, namely the generalization of the nearly-symmetric kinematic model described earlier. Here the prevailing field and flow are assumed to be predominantly axisymmetric and azimuthal, and the container is supposed to rotate with large angular velocity $\boldsymbol{\Omega}$. Clearly, the first two anti-dynamo theorems apply, and regeneration of this toroidal field will not be possible unless the symmetry is broken and/or poloidal motion is provided. Braginskii (1964*b*, 1967*a*, *b*) proposed, however, that the basic state is dynamically unstable to asymmetric wave-like disturbances, which progress in longitude round the rotation axes. The dynamo feeds its Joule losses from this instability. This idea is not unrelated to the role proposed by Gilman

(1969*a*, *b*) and Gordon (1972) for baroclinic instabilities of a zonal flow, which Gordon described to the meeting.

Braginskii's theory was discussed and developed by *D. J. Acheson* and by *J. R. Booker* at the meeting, and some of the attendant difficulties were highlighted by *K. Stewartson*; published work has been recently reviewed by Hide & Stewartson (1972) and by Roberts & Soward (1972). Some support for the existence of these large scale waves in the sun was given by *S. T. Suess*. Hide (1966) has long argued that the westward drift of the geomagnetic field is a manifestation of a wave motion; see §5 above.

For large values of $\Omega\mathscr{L}/V$, where $V = \mathscr{B}/(\mu\rho)^{\frac{1}{2}}$ is the Alfvén velocity based on the toroidal field strength $\mathscr{B}$, the waves divide into two groups; fast inertial waves and slow MAC waves, where MAC stands for Magnetic–Archimedian (buoyancy)–Coriolis, after the forces which determine the propagation of the waves. It is the MAC waves which are often thought to provide the necessary α-effect. Nevertheless, it can be demonstrated that, in the state of neutral stability, α vanishes when every MAC wave carrying energy outward is balanced by a similar MAC wave carrying energy inward. In both the Moffatt and the Braginskii approach, the fundamental reason for the asymmetry in wave propagation is lacking. Absorption of energy in the Ekman–Hartmann layer at a solid boundary appears to be too weak. Is critical layer absorption indicated, or will gradients of mean density provide the explanation? Clearly, considerable effort will be expended in answering this question in the future.

The theory of MAC wave propagation can be understood on a theory in which inertial and viscous forces are discarded, to give 'the slow-steady equations'. These are subject to a certain consistency condition, first given by Taylor (1963). When this is satisfied, the equations may be solved for $\mathbf{u}$ to within a motion constant on cylinders coaxial with the rotation vector. Even this may be obtained by differentiating the consistency condition by time and applying the induction equation. *R. Thirlby* described briefly his inconclusive attempts to solve the slow steady equation and the induction equation simultaneously. Plans for a similar study were announced by *G. O. Roberts*, who showed that the time step for integration must be short compared with the period of the MAC waves relevant to the grid selected. He noted that this was likely to be a stringent condition in the numerical work.

F. H. Busse, *S. Childress* and *A. M. Soward* described simple layer models of thermally driven MHD dynamos. The presentations were independent, though similar in content; in particular, the approaches of Childress & Soward (1972) were very closely related. In each case, the field represented a small (but finite amplitude) disturbance superimposed on a highly rotating fluid layer close to a state of marginal convection (e.g. Chandrasekhar 1961). As Childress observed, this permitted a direct application of the two-length scale method, for it is well known that the preferred convective cells are on a scale L of order $T^{-\frac{1}{6}}\mathscr{L} = \epsilon\mathscr{L}$, say, where T is the Taylor number, $4\Omega^2\mathscr{L}^4/\nu^2$. The critical Rayleigh number R_c for convection is of order $T^{\frac{2}{3}} = O(\epsilon^{-4})$. Thus, if the magnetic Reynolds number R is $O(\epsilon^{\frac{1}{2}})$, and the limit $\epsilon \to 0$ ($T \to \infty$) is assumed, the dynamo problem for the layer becomes simple, in principle. Marginal regeneration is found to occur, as an a.c. dynamo wave, provided the plan-form of the convection is not a roll; the most straightforward case is the rectangle, i.e. two perpendicular rolls of different amplitude. When R exceeds the critical value, the field grows until a state of balance is struck. The case in which the resulting Hartmann number $B\mathscr{L}(\sigma/\rho\nu)^{\frac{1}{2}}$ is of order unity was given particular attention by Soward in his talk.

The Childress–Soward model is governed by three partial differential equations, first order in t and second order in z, the vertical co-ordinate. These bear a striking formal relationship to those governing the self-reversing disk dynamos of Rikitake (1958; see also Cook & Roberts 1970), and it was perhaps here that theoreticians came closest to taking up the challenge of Bullard in the first lecture to the meeting, namely that of providing a rational explanation for reversals of the geomagnetic field.

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